Problem Solving Through Problem Posing: The Experience of Two Teacher Education Students

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Teacher education students in their first year of university were invited to take part in an eight week program involving solving problems and posing problems on the basis of the given problem. Data concerning two of the eight students who agreed to participate, have been reported in terms of their problem solving strategies, their progress in problem solving skills and their expressions of cognitive and affective factors related to the study. It was found that the requirement to pose a similar problem to a given problem helped clarify thinking and that other important factors were the realism of the problem, the capability for visualisation and the nature and speed of feedback.

Despite over a decade of interest and emphasis on problem solving as a strategy for teaching mathematics and as a life skill, or perhaps, because of it, problem solving in schools appears to either be viewed sceptically as too hard or in an unenlightened way as being fairly routine. Indeed, many teachers who were quite excited about the possibility of incorporating problem solving into their daily classes in a way that would grasp the imagination of the students, have lapsed back into the habit of treating problem solving in a routine way. Whichever situation occurs, it is disappointing to note that the approach which held so much promise, challenge and excitement has reverted to the chore it was always considered to be. Even teachers and students who have maintained their enthusiasm for the use of problem solving as an important skill, sometimes feel overwhelmed and lack the confidence to explore the art of problem solving creatively. This can be extremely frustrating to the student and cause loss of self-esteem in the teacher.

Problem solving as an essential element in school mathematics has been advocated since the early 1980s by such seminal documents as the *Agenda for Action* (National Council of Teachers of Mathematics, 1981), the Cockcroft Report (1982) and the *National Statement on Mathematics for Australian Schools* (1989). The best indication of the framework within which this emphasis was to be implemented appeared in the report of a national workshop in relation to the Australian Mathematics Education Project (1981) and has been used in several states in their curriculum documents and other professional development activities. This framework lists three different aspects of teaching problem solving in schools, viz. teaching <u>for</u> problem solving, teaching <u>about</u> problem solving and teaching <u>through</u> problem solving. Teaching for problem solving means ensuring the learner acquires the skills, understandings and knowledge necessary for the task of problem solving. Teaching about problem solving means ensuring the learner has at his or her disposal the strategies or heuristics necessary for problem solving. Teaching through problem solving means using problem solving as the methodology through which mathematics is taught. Resources and research have tended to focus on the first two of these aspects. The third has been considered quite difficult except perhaps in the early years of schooling. One example of a positive result in teaching through problem solving is given by Villasenor and Kepner (1993). They used two groups of first-grade teachers and their classes. One group of teachers taught their classes using a problem solving approach and the other was a control group. They found that the experimental group performed better in problem solving than did the control group. This result seems to indicate that teachers who make the effort and are appropriately supported can make a difference in their students' problem solving ability. The researcher considers this result needs to be extended to problem posing as a complement to problem solving and therefore a possible tool for achieving the use of problem solving as a methodology for teaching mathematics.

In considering the reasons for a regression to routine treatments of problem solving in schools, the researcher became aware of two factors. One was the solver's lack of success in the problem solving process and the other was the solver's lack of willingness to attempt new methods and strategies. It was conjectured that the latter difficulty might well

be overcome by students becoming more adept and creative in the solution of problems through the medium of posing problems.

The obverse side of problem solving is problem posing, and again, there have been several (Skinner, 1990, Brown and Walter, 1990) in the past decade who would herald problem posing as a panacea for deficiencies in mathematical thinking. Sometimes this is related to the notion of mathematical investigations in which learners are asked to pose problems about a particular situation, which is either well structured or loosely structured or having no structure at all, if that is possible. Other times, learners are given no assistance at all and are asked to pose problems on a fairly random basis. There are several other variations of the use of problem posing as a means of developing mathematical thinking and therefore of enhancing problem solving. Silver, Mamona-Downs, Leung and Kenney (1996) studied the problem posing ability of middle and prospective secondary school teachers in connection with a complex situation before, during and after attempting to solve a problem within the same setting. They found the subjects generated a large number of varied problems, more before the solution process than during or after. Another study by Silver and Cai (1996) asked middle school students to write questions in relation to a story situation that could be answered from the information given, This also produced a large number of problems though not all solvable. This same study also looked at the linguistic and mathematical complexity of the problems posed.

Ellerton and Clarkson (1996) claim that research in this area suffers from a lack of a research framework and give an account of the types of research that have been carried out in the last fifty years on problem posing. Much of the emphasis seems to have been on the development of skills in problem posing in a scientific sense - looking for the questions one should be asking to carry out appropriate research into a problem (p. 1010). They do, however, stress the importance of problem posing as a skill. Brown and Walter (1990) take it a stage further. They claim that there is a direct link between problem posing and problem solving in that problem solving produces problem posing. English (1997) investigated the problem posing abilities of third grade children and found that they had a limited range of problems posed which did not change greatly with their participation in a problem posing program. The program did, however, improve their problem posing skills.

In a paper presented at a previous MERGA conference (1995, cited by Ellerton and Clarkson, 1996), Stoyanova suggested a three-way categorisation of problem posing under the headings *free*, *semi-structured* and *structured*. A *free* problem posing situation is one in which the learners are asked to pose a problem from a given situation or for a particular purpose. A *semi-structured* problem posing situation is one in which 'students are given an open situation and are invited to explore the structure and to complete it by applying knowledge, skills, concepts and relationships from their previous mathematical experiences' (Ellerton and Clarkson, 1996, p. 1011). A *structured* problem posing situation is one in which the learners are asked to pose problems based on a specific problem. It was this third category of problem solving that the researcher explored.

Observation has indicated to the researcher that many teacher education students are not enthusiastic about mathematics as a subject and problem solving in particular, and, in fact, experience a great deal of anxiety over the subject. This may be because of previous lack of success in mathematics and the tendency to rote learn, a method which is not as successful in problem solving as it might appear to be in other areas of mathematics. It may be that this lack of success in the past has conditioned students to react negatively to any aspect of mathematics and has lowered their self-esteem, thus making it difficult for them to take the kind of risks that may be necessary in problem solving, This is particularly true, it is felt, on the part of girls and women, and so relevant for teacher education in that the majority of primary teacher education students are female,

Problem posing is seen to be more akin to the way in which women operate and learn (Brown, 1984) so extending problems to be solved into the task of posing problems should suit female learning patterns. The researcher, accordingly, considered that an exploration of the effects this strategy, i.e. extending problem solving to problem posing, would have on beginning teacher education students, would add to the knowledge already growing concerning problem posing. The researcher explored the complementary procedure of problem posing with a view to ascertaining whether experience and hopefully success, in

problem posing would assist students in problem solving. While, in general, problem posing can cover a broad range of possibilities, it was thought that, initially, this study should be confined to one of the simplest forms of problem posing, and that is, the posing of similar problems to given problems. This approach was taken because it was felt it would suit students with little or no experience in problem posing by not expecting them to stretch themselves too far. Because of the problem given, students would have a model and a structure which they could use in posing a new problem. It was on this basis that this study was structured, subjects were sought and the instrument devised and the research plan implemented in an attempt to explore the research question.

The Research Question

Will experience and skill in posing problems related to a given problem help pre-service teacher education students become better problem solvers?

The Subjects

The subjects were eight pre-service teacher education students in their first year of training. Volunteers were sought, as the group was large in number and it was considered that those who wanted to be involved would provide the most useful information. Twenty volunteers were sought but only fourteen students agreed to take part after they had been fully briefed on what was involved. Of these only eight continued through the whole process so the numbers fell far short of what was thought to be needed for a worthwhile study. This drop off possibly occurred for several reasons. One was pressure of work for students who were feeling insecure in some of their subjects. Another was that the study extended over two semesters and in the second semester, the researcher was not involved in the subjects they were taking. This meant that keeping contact became very difficult. Subsequently, however, the information gained from the participation of the students, even those who did not complete the sequence of problems, proved to be valuable and therefore analysed for comment. For this report, however, data concerning two subjects only will be presented, they being the ones on whom the most complete information is available.

Of the students who volunteered to participate, only two felt confident about their ability to do mathematics. Most were very anxious about the subject and several gave, as their reason for volunteering, their hope that the project might make them better at mathematics because they realised they would be teaching it. Their background differed, too. Only one had completed Level 3 in the Higher School Certificate and several had not done mathematics since they were in Year 10 or its equivalent in the case of mature age students.

All the volunteers were female. Only one male showed an interest in the project but did not eventually take part. The lack of male participation was not surprising though it was disappointing. The overwhelming majority of teacher education students in that year's intake are female.

The Instrument

The instrument consisted of 8 problems selected from *Teacher Tactics for Problem Solving* by Stacey and Southwell (1996). Three problems were of a general nature, three were numerical and then there were two that involved spatial concepts as well numerical concepts. Two additional questions were added at the request of the students who expressed a desire to have more practice in geometry. The problems were fairly elementary but lent themselves to good extensions and more than one strategy or answer, thus giving students the opportunity to demonstrate higher order thinking. Only one problem could be classed as a routine problem in the sense of requiring only the application of an algorithm while a part of one other problem was routine. The rest of the questions were non-routine. Also, all except parts of two questions required a number of steps for solution. The problems used are summarised in Table 1.

Table 1. Characteristics of Problems

Problem	Type	No. of Steps	Topic	No, of Possible Methods
1(a)	routine	2	money	1
1(b)	routine	3	money	1
1(c)	non-routine	-	money reverse	2
2	routine	3	money	2
3	non-routine	-	party tables	4
4	non-routine	-	tournament	3
5	non-routine	-	toaster	2
6	non-routine	-	chocolate bar	2
7	non-routine	-	cafeteria roster	2
8	non-routine	-	strip of paper	4
9	non-routine	-	diagonals of polygons	4
10	non-routine	-	exterior angles of polygons	4

Methodology

Each participant was issued with a small exercise book in which the first problem was printed. They also received a printed copy of the task to be carried out each week in relation to the given problem. They were asked to divide a double page in their exercise book as follows in Figure 1 to carry out their weekly task:

Week	Problem Solution	Problem Posed and Solved	Comment

Figure 1: Layout for Weekly Assignment

The task consisted of:

1. Solve the problem.
2. Pose a similar problem and solve it.
3. Write a comment on the problem given, the problem posed, or any other relevant matter.

This procedure was explained carefully to them in the first meeting between the participants and the researcher. Students did need, however, frequent reminders concerning this procedure and these were provided by the researcher in writing.

The participants were asked to hand in their book at the lecture each Monday at which time they would receive the next problem. This worked fairly well during the first semester of the year but when the researcher no longer took them for the lecture, it was more difficult to monitor progress. The books were marked each week and appropriate comments and suggestions were made. They were then photocopied and handed back to the student. This process continued for the duration of the study.

After participants had been working on the problems for four weeks, each one was invited to an individual interview with the researcher who had a prepared list of questions to

ask. Some participants preferred to write their own answers so they would have time to think about the questions. Others were content to answer the researcher's questions so that she could record them. This process was repeated with the same set of questions at the conclusion of the study, i.e, when the participants had completed 8-10 problems. On one occasion, two participants asked could they be interviewed together and this was done. The questions asked were:

1. What is problem solving?
2. What did you find easy and what was hard about the problems you have been asked to solve?
3. Was having to pose a related problem a help in actually solving problems?
4. What use to you was the commentary you wrote on your problem solving and problem posing?

Analysis

The data collected consisted of the participants' solutions to the given problems, the problems posed and their solutions, the comments of the students each week and the responses to the two questionnaires. These were all analysed by the researcher and common elements extracted. Very different views were also noted and reasons for these sought. Particular attention was given to problem solving strategies and any changes in approach or skill that took place over the time of the study.

Results

The results are presented for two subjects in the study. Each section is described according to the data collected and then an attempt to provide a synthesis of them is made. As indicated above, the students were asked to solve a problem each week, pose a similar problem, solve that, and write a comment on the procedure.

Joyce: Joyce was able to get the right operation and answer for the routine question and for the routine sections of one other question but did not set out her work in a logical fashion. When she posed a similar problem, she changed the context and the numbers and, in one case, kept the numbers in the same ratio. This is an acceptable method of generating a further problem from a problem (Randall and Lester 1982). For the non-routine questions, Joyce used either a diagram, made a list or listed all possibilities as her main strategies and sometimes used two strategies together. On one of the problems she arrived at an answer which was obviously unrealistic but there was no attempt to check it. On another two problems, Joyce either misread the problem or did not examine the problem to see what it was really asking. She did admit that she was not always sure what was wanted. In regard to the first question, which had a non-routine third part, Joyce was able to pose a similar problem but admitted that after she had given a solution, she looked at it again a week later and changed her answer, at the same time saying in her commentary that she was desperate. Another indication of her lack of confidence was her several false starts and finishes. Work was crossed out fairly frequently and answers or methods questioned. She did, however, on the fifth question, begin to play around with the problem statement and explore some different interpretations and strategies. She only attempted to pose similar problems for two of the non-routine problems. This was unfortunate in light of the objective of this study.

Joyce's commentary on her work was reasonably full and apparently frank. On four of the problems, she indicated that she had either changed her thinking about the solution or would have liked more information. She also wrote some comments which were quite self-deprecating. In relation to the first question with the third non-routine part, she finished with "I can't believe I didn't see it earlier". This remark is even more poignant when one realises that she had not solved the problem after all. Another comment was "I don't like this much" then she questioned the use of the type of toaster in the question. Her solution (also expressed by other participants in the study) was to buy a popup toaster. On three of the problems she made reference to the usefulness of the problem. This seemed to make their solution more desirable and therefore she seemed more motivated to attempt the solution.

One other critical characteristic that emerged from the commentary was Joyce's preference for visualising through diagrams or writing lists or matching possibilities.

Answers to Question 1 in the questionnaire changed a little in the course of the study. From "finding an answer to something quite difficult" to "working out the answer to something that is not obvious at first" seems to imply that Joyce's view of problem solving changed from the main characteristic of difficulty to that of unfamiliarity or some sort of barrier. In the earlier interview, she pointed out that if there is an easy answer to a question, there is no problem.

Joyce stated in the first interview after attempting four questions that, with the exception of the third part of the first question, she found the questions relatively easy. She admitted, however, that she had trouble 'seeing' the problem. This was an interesting statement in that she found the correct answer almost immediately, then experimented with several other possibilities and finally opted for one which did not fit the specifications of the problem. In the later interview, she simply stated that any problem for which she could use a diagram, she found easier than the others.

Posing problems initially was not perceived as being helpful. Joyce did state, however, that if she was to do the task again, she "could see this might be of value" and she would use the tactic if she reached a standstill. This means she saw problem posing as a means of solving a problem but not necessarily as a strategy for her to develop her problem solving ability. In the later interview her comment was simply "helped to clarify my own thinking".

Joyce's comments in reply to the fourth question were as follows:
"I loved having a chat with you, B..... and it did help me clarify my thinking".
"It was good to be able to let you know how I felt about the question and explain how I went about it. I was more forthcoming in the comments because I knew and liked B..... Feedback is good and it is very important for me to read it".

Matilda: Matilda proved to be a much more confident and adventurous problem solver. Right from the first question she tried several different strategies for the non-routine questions. For the routine questions, she was able to find the correct answers without difficulty, set her work out logically, explained why she did each step and posed problems using a different context and different numbers. She attempted algebraic strategies even for routine problems and checked her work in several problems. She used several strategies in her solution of non-routine problems, including logical deduction, diagrams, listing all possibilities, making lists and looking for patterns. The problems posed were either using a different context or extensions of the original problem to develop generalisations. In one case, the same context was used but different numbers.

The commentary written by Matilda was more often than not interposed between steps in the problem solution. It contained frank admissions of frustration at least once and failure on two occasions. "There must be a way". Most of the other comments had to do with Matilda's thinking process and further clarified how she arrived at solutions, particularly where generalisations were concerned. The posed problems were all solved and in several cases led to further generalisation. She also suggested the solution to the toaster problem was to buy a popup toaster. It appears that the practicality of making toast got in the way of the mathematical problem this question posed.

Matilda's idea of problem solving was expressed in quite different terms from one interview to the next. In the first interview, she said "'Problem' has a negative connotation to me. Before I read it, I always think it will be hard just because it is a problem". In the second interview, her words were "Problem solving is finding an answer to something that is hidden at first - something you can't see as soon as you look at it. If it is a problem for one, it may not be for another". Like Joyce, Matilda recognised the relativity of problems.

In answering the second question on the questionnaire in the first interview, Matilda said she found the third part of the first question hard but the others were easier. She had spent a long time on it. She also said she used algebra "more than [I] probably should". In the second interview, her response was: "At the end they were easy. I got better. I found things hard at the beginning. I learnt how to think what it meant, how I thought about it. Once I knew what to do, doing it was easy."

Making up another problem helped Matilda mainly by confirming that she had done the first problem correctly. This was Matilda's response to the third question on the questionnaire in the second interview. This view seemed even stronger in the first interview in which Matilda responded by saying: "I find that posing another problem always helps me. It verifies that problem that I have already done and gives me more confidence for the next one." It may be that the boost in confidence was taken for granted by the time Matilda reached the eighth or ninth problem.

The commentary was also considered helpful. "It makes me think about what I am actually doing. How I am exactly going about solving the problem. Feedback is definitely necessary". This view was re-inforced in the second interview in which Matilda also commented on the value of talking to the researcher in the commentary or by phone and knowing she would always respond.

Discussion

The two teacher education students whose work and understandings have been reported are both mature aged women with different personalities and mathematical backgrounds. From their own reports through the interviews, both gained from the experience of posing problems similar to given problems, though one less so than the other. In this regard, it would appear that the process is better exploited by individuals with some degree of mathematical knowledge and confidence than by those with a less adequate mathematical preparation and less confidence.

Both subjects used a number of strategies, again with a differentiation between them according to their backgrounds and personality. More strategies were used by the students with the more adequate background. As neither had previously had any formal work on problem solving in their university course, it can be assumed that the strategies used were natural to them. Two further aspects of the strategies used relate to the content of the problems themselves. Problems that students perceive as having a practical use appear to be more worthy of solution in the eyes of the students and therefore they are more motivated to find a solution. In one case, however, the subjects' previous practice hindered their solution to the mathematical situation portrayed in the problem. The other relevant factor was whether a strategy that involved some form of visualisation such as a diagram could be used as part of the strategy. This preference shown by one subject in particular may be an aspect of learning style or it could be that the problems chosen all had easier solutions using this strategy.

The kind of relationship developed between the subjects and the researcher indicates that students respond well to the opportunity of being able to discuss their solution attempts and their frustrations with a person who can be relied on to listen and respond. This person could be a teacher but could just as well be a peer. The discussion can be spoken in person or on the phone, in writing or some other way. In this study, most of the conversation was through the interviews and through the commentary written by the subjects to which the researcher responded. The opportunity to record thought processes and feelings as a result of solving a problem proved helpful to both subjects. They gave two main reasons. These were that it clarified their own thinking and they were able to explain what they had done. It would be foolish to think that all students would find the process of recording their thoughts and feelings when they have solved a problem easy or helpful. Many would not and so it is necessary to develop some techniques which provide the incentive and the practice students needs to enable them to use this commentary effectively. Some possible techniques are:

- Ask orally a very straightforward question on the problem then get the students to record their answer. (one sentence at first, then two sentences, etc.)
- Provide the students with a problem solution. Ask them to write down whether they agree with the solution and why.
- Devise a game in which students draw a problem from a stack of problem cards. Each student presents their problem and ask the rest of the class to solve it and write an explanation of their solution strategy. Start with routine problems requiring one step only and increase complexity over time.

Feedback from the researcher also seemed to be important to the subjects. This finding re-inforces the view that the more feedback can be given and the faster it can be given, the more helpful it is to the learner.

Conclusion

On the evidence provided by the performance of the two subjects described, it is possible that posing problems based on given problems can be a useful strategy for developing the problem solving ability of teacher education students preparing to teach mathematics in the primary school, more so with the more mathematically able students than with those with a less adequate background in mathematics. Other factors of significance seem to be the practical nature of the problem and the ability to use strategies which involve some form of visualisation. The relationship with a reliable discussant and the frequency and speed of feedback are also important.

The findings of this study would be enhanced by confirmation or otherwise resulting from a repetition of this study with an increased number of subjects. As well, this study raises several issues worthy of further study. These include the role and forms of motivation in problem solving and problem posing and the relationship between problem solving, problem posing and personality. As problem solving is an important goal and process of mathematics education, these and related matters need urgent investigation.

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